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# REVIEWS

**Thermo-fluid Dynamic Theory of Two-phase Flow.** By M. Ishii. Eyrolles, 1975. 248 pp. 83F or $21.60.

In this book the general equations governing two-phase flow are developed, the foundations of the two-fluid and mixture models are discussed in detail and the simplifications appropriate in certain special cases are indicated. The detailed calculations are carefully set out, and I detected very few slips in the mathematical results. The same cannot be said of the grammatical construction (in particular, the relationship between the number of subject and verb seems quite random), but one soon becomes used to the author's idiosyncratic use of English, and it is no real impediment to understanding.

The treatment is limited to two-phase flows, although certain results incorporate summations that could readily be extended to further phases. The systems of primary interest to the author are continuous fluids containing drops or bubbles (the effects of surface tension and phase change are included systematically), but most of the results apply also to motions involving solid particles. The continuous phase is taken to be a pure substance, no allowance being made for molecular diffusion through it, although reference is made to the parallels between such microscopic diffusion and the 'diffusion' of finite particles.

In a monograph of this kind one would expect an authoritative list of references, and it is surprising to find that such names as Batchelor, Brenner, Gal-Or, Jackson and Marble do not appear. Also missing from the references are a few items cited in the text.

A good deal of space is devoted to a discussion of various kinds of averages, but the idea of ergodicity never appears explicitly. A feature of the treatment which puzzled me was the adoption of time averaging as the basis of the analysis, rather than the volumetric averaging more often used. Even after a conscientious attempt to understand the author's point of view, I cannot feel that his choice of technique is judicious. The analysis of situations that are inherently difficult to visualize is not helped by considering isolated elements on the surface of myriad particles rather than the complete surface of an individual particle. Moreover, the intervals over which one sums are not intrinsic to the structure of the medium, but are dependent on the velocity of convection relative to the observer. This limitation is particularly apparent if one seeks to encompass the important limiting case of flow through a stationary matrix (a porous medium) into one's analysis of two-phase flows. Similar problems arise when considering situations in which part of the field contains rigidly locked particles, while other regions contain flowing dispersions, as for example, in an eroding stream bed. These conceptual difficulties are mirrored in the detailed analysis; Dr Ishii's discussion of the averaging of derivatives is far from clear, and little indication is given of the role of spatial variations of averaged quantities. In short, the technique of time averaging adopted in this book introduces

difficulties in both understanding and mathematical representation. Moreover, in most applications one must ultimately assume ergodicity, as does the author tacitly.

I am conscious of presenting a somewhat negative view of Dr Ishii's work. There are in fact few systematic expositions of the foundations of what may be termed the engineer's theory of two-phase flow, and workers in the area will welcome this largely successful attempt to fill the gap.

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**Finite Elements in Fluids.** Edited by R. H. GALLAGHER, J. T. ODEN, C. TAYLOR and O. C. ZIENKIEWICZ. Wiley, 1975. Vol. I, 290 pp. Vol. II, 287 pp. Each £9.75.

These volumes grew from the *International Symposium on the Finite Element Method in Flow Problems*, held at the University College of Wales, Swansea, in January 1974. As noted in the volumes' prefaces, the symposium provided a forum for three different types of paper: invited state-of-the-art reports; in-depth treatment of particular problems; and finally outlines of work complete or in progress on particular problems. The symposium proceedings contained full-length papers in the last category plus extended abstracts of papers in the other two categories. The present books contain full-length papers from the first two categories. The two volumes are subtitled *Viscous Flow and Hydrodynamics* (volume I) and *Mathematical Foundations, Aerodynamics and Lubrication* (volume II).

Volume I begins with an introduction to the finite element method, followed by three chapters on viscous flow, three chapters on the analysis of large bodies of water and three chapters on groundwater flow, and concludes with four chapters on hydrodynamics and wave problems. In volume I we find papers which, in addition to being overviews, generally provide a detailed discussion of the fundamental equations, various variational or weighted-residual methods and the elements used, and specific example calculations. Most of the works deal with simple finite elements of the rectangular and triangular type using linear, quadratic and higher-order polynomial fits. In all but two cases the elements used are $C^0$, i.e. allow continuity of the unknown function but not its derivatives across element boundaries. An impressive and interesting array of physical problems is discussed.

Volume 2 contains seven chapters on mathematical foundations, three chapters on the general area of aerodynamics, two chapters on lubrication, and a chapter on the special topic of hydromagnetic stability studies. As with the first volume, the reader can find the scope and focus of the volume described precisely in a detailed preface. Whereas the first volume focused on specific equations, methods, elements and problems, the second volume is much more sketchy with regard to the actual finite elements used and gives very little detail on specific examples. The mathematical foundation papers are highly theoretical and often discuss, with equal weight, finite element methods and other techniques, related or unrelated to the finite element technique. The chapter on

numerical analysis of transient and nonlinear waves, while giving an interesting formulation and solution technique, is not really related to the finite element method and seems somewhat out of place in this volume. Interestingly, the chapter on computation of flows in turbomachines is a review of various methods in use without including any from finite element technology; indeed, the author suggests some specific limitations which the finite element method must overcome if it is to find its place in the repertoire of turbomachine flow calculations.

These two volumes offer a group photograph at a point in time of a very rapidly moving field: herein lies their strength and weakness. They do offer an exceptional coverage of the field in terms of problems described, equations employed, variational and other mathematical bases, and the finite element techniques which are used. However, in general, the field has moved well beyond these works. This leaves us with a good overview, circa 1974, but neither a current state-of-the-art report nor an in-depth textbook. The most significant weakness of the volumes is a lack of coverage on the so-called 'higher-order' finite elements, which are now coming into their own. These elements offer important continuity properties and simplifications of calculations, as well as a significant reduction in the number of elements required. Thus we speculate that the solution offered in chapter 14 of the first volume for the finite element analysis of jet impingement on axisymmetric curved deflectors could be accomplished now much more accurately and simply with higher-order elements. These volumes, then, would be a valuable addition to institutional libraries, useful for students and those seeking an overview, but not volumes that one could recommend for a personal library.

Finally, these volumes are not really for beginners in any field. Serious readers should come to these volumes with a fundamental knowledge of finite element techniques obtained from books such as those by Huebner, Zienkiewicz or Strang and Fix, plus an understanding of the governing mechanisms of the physical problem to be examined.

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